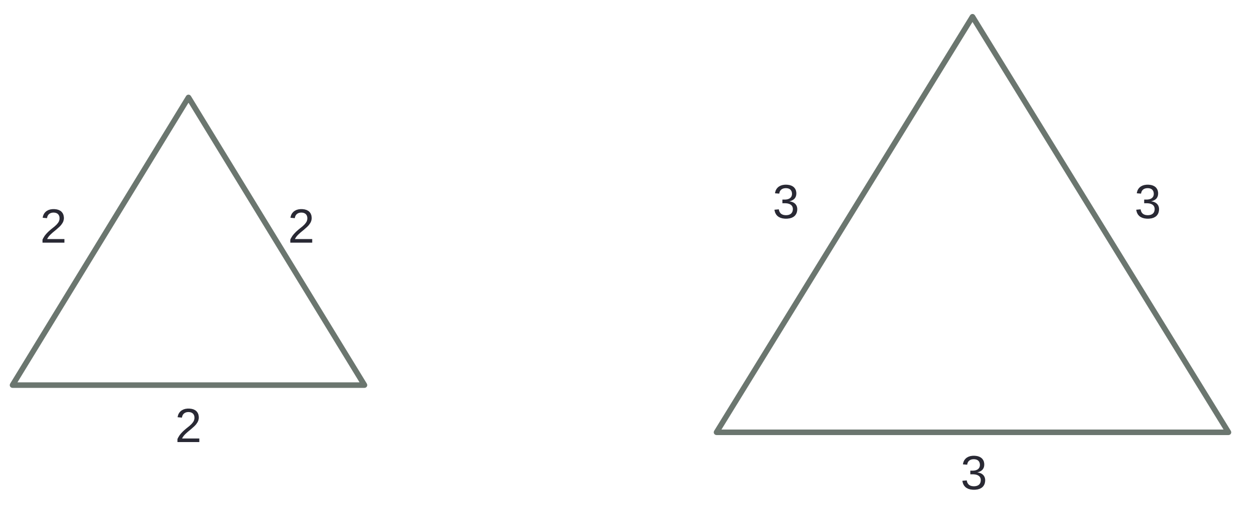
RATIOS, PROPORTIONS, AND THE GEOMETRIC MEAN

"A life not lived for others is not a life worth living." –Albert Einstein

Concept 1: Ratios

- Ratio-2 numbers that can be compared and $b \neq 0$. Ratios are written as **1:2** or **ratio of 1 to 2** or $\frac{1}{2}$.



Concept 1: Ratios

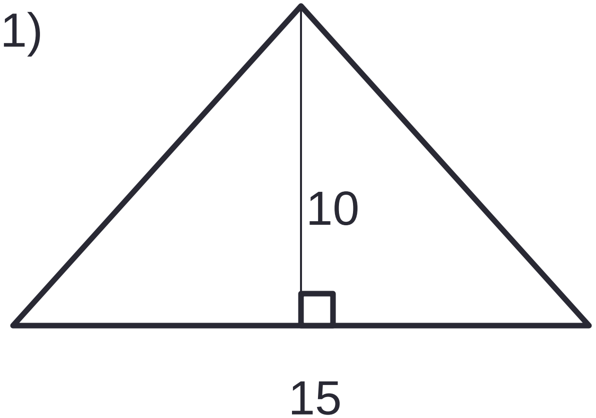
- Ratios can be simplified. If all parts of the ratio are a multiple of the same number, then you can divide all parts of the ratio by that number.

- Example: 8:16 can be written as 1:2
- Example: 6:18 can be written as 2:6 or 1:3
- Example: 8:10 can be written as 4:5

Example 1

- Find the ratio of the base to the height of each triangle. Simplify the ratio, if possible.

1)



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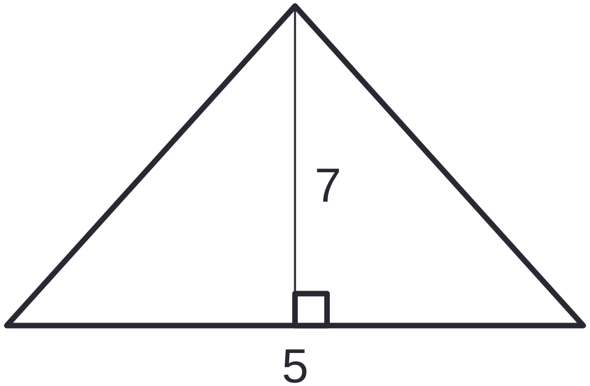

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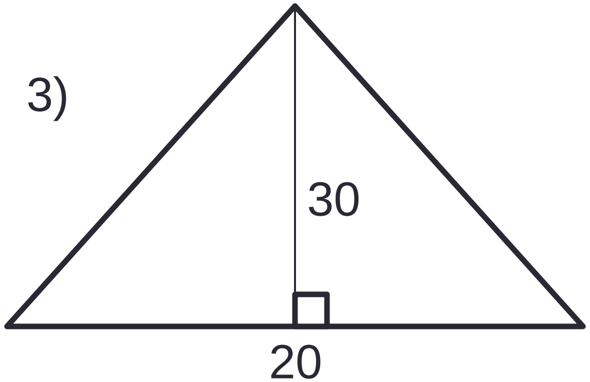


Example 2

- Let x=5, y=9, and z=15. Write the ratio in simplest form.

1) x:2z

2) $\frac{y+3}{z}$

3) $\frac{xy}{z^2}$

Example 3

- The measures of the angles of a triangle have the extended ratio of 2x:3x:4x. What is the measure of the angles?

Concept 2: Proportions

- An equation that states two ratios are equal is called a proportion. $\frac{a}{b} = \frac{c}{d}$ is an example of a proportion. The means of the proportion are b and c. The extremes of this proportion are a and d.

- Example: $\frac{5}{6} = \frac{18}{x}$
  - 5 and x are the extremes of this proportion.
  - 6 and 18 are the means of this proportion.

Concept 2: Proportions

- To solve a proportion we can use a property of proportions.

1) Cross Product Property: In a proportion, the product of the means equal the product of the extremes.

Example: $\frac{5}{6} = \frac{15}{18}$ ➔ 5(18)=6(15) ➔ 90=90

Example: $\frac{5}{6} = \frac{18}{x}$ ➔ 5x=6(18)

Example 4

- Solve for the variable.

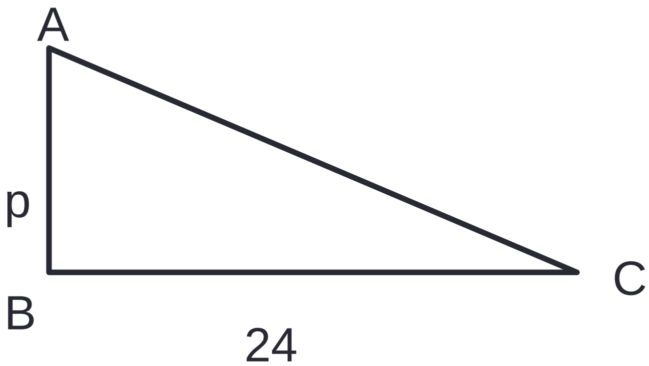
1) $\frac{3}{5} = \frac{15}{x}$

2) $\frac{6}{16} = \frac{x}{8}$

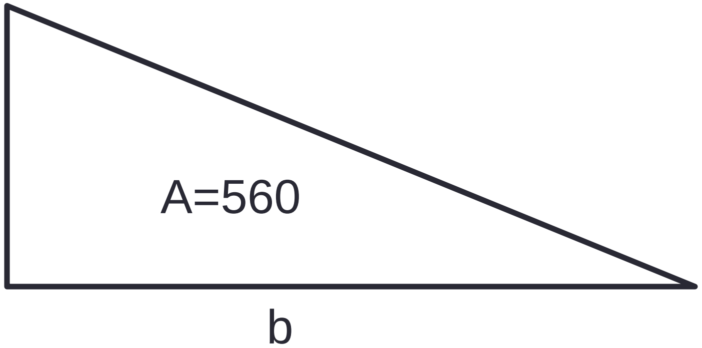
3) $\frac{3}{2p+5} = \frac{1}{9p}$

Example 5

- The ratio of two side lengths for the triangle is given. Solve for the variable.



AB:BC is 3:8



Base to height has a ratio of 5:4

Concept 3: Geometric Mean

- The geometric mean is a length that can be constructed using properties of triangles. For now, the way to find its value is through proportions. The geometric mean is the square root of the extremes of a proportion.

- Example: $\frac{a}{x} = \frac{x}{b}$ ➔ $x^2 = ab$ ➔ $x = \sqrt{ab}$
  - x is the geometric mean.

Example 6

- Find the geometric mean of:

1) 24 and 48
    1) $x = \sqrt{(24)48}$
    2) $x = \sqrt{(24)(24)(2)}$
    3) $x = \sqrt{24^2} \cdot \sqrt{2}$
    4) $x = 24\sqrt{2}$
2) 12 and 27
    1) $x = \sqrt{(12)(27)}$
    2) $x = \sqrt{(3 \cdot 4)(3 \cdot 9)}$
    3) $x = \sqrt{(9 \cdot 9)(4)}$
    4) $x = \sqrt{(9^2)} \cdot \sqrt{4}$
    5) $x = 9 \cdot 2$ or 18

USE PROPORTIONS TO SOLVE GEOMETRY PROBLEMS

“Bad is never good until worse happens.” – Danish Proverb

Concept 4: Proportion Properties

1) Cross Product Property
2) Reciprocal Property-If two ratios are equal, then their reciprocals are also equal.
3) If you interchange the means of a proportion, then you form another true proportion.
4) In a proportion, if you add the value of each ratio's denominator to its numerator, then you form another true proportion.

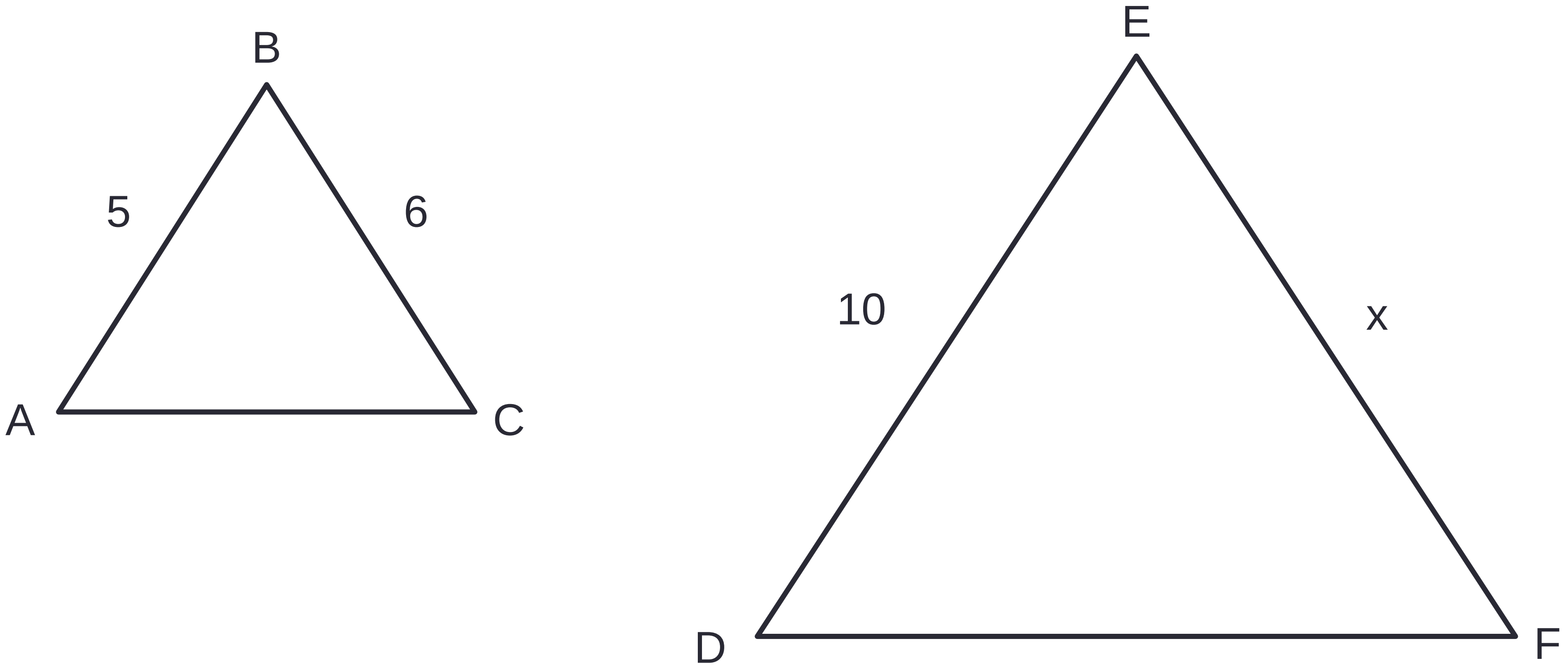
2) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$

3) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$

4) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$

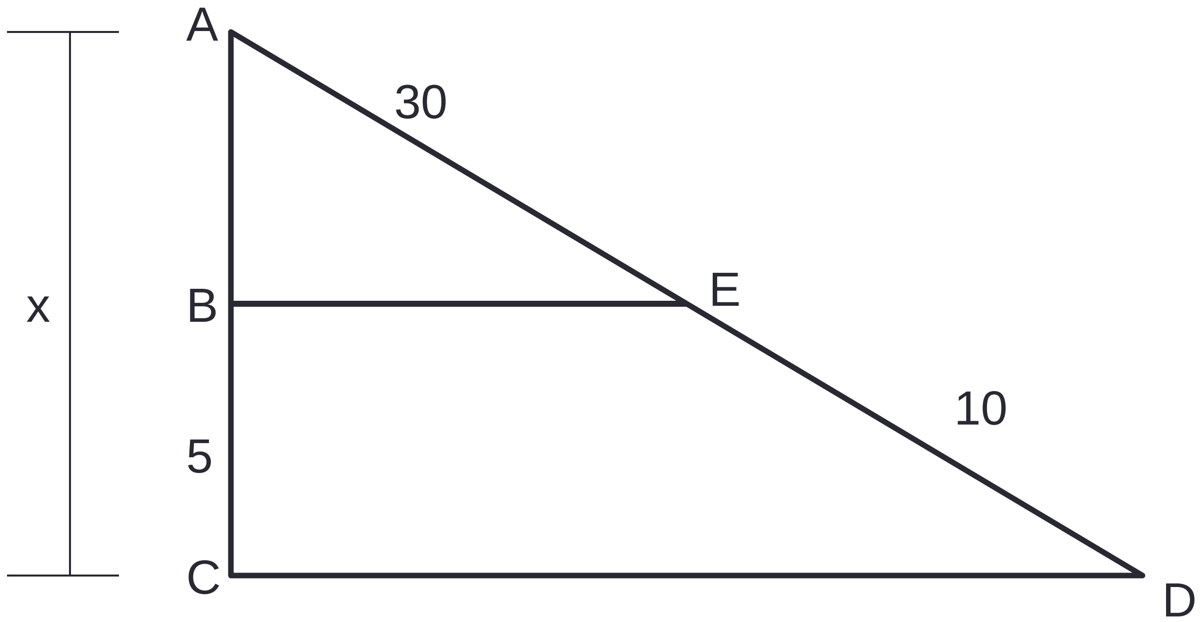
Example 1

- In the diagram, $\frac{AB}{BC} = \frac{DE}{EF}$. Write 4 true proportions.



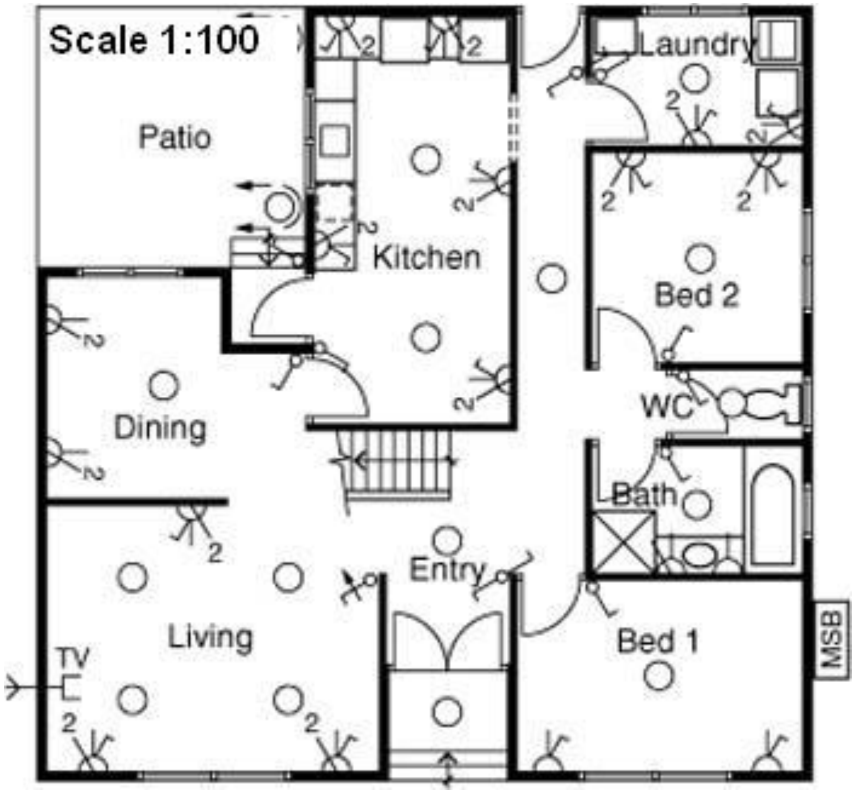
Example 2

- In the diagram, $\frac{AB}{BC} = \frac{AE}{ED}$. Find AB and AC.



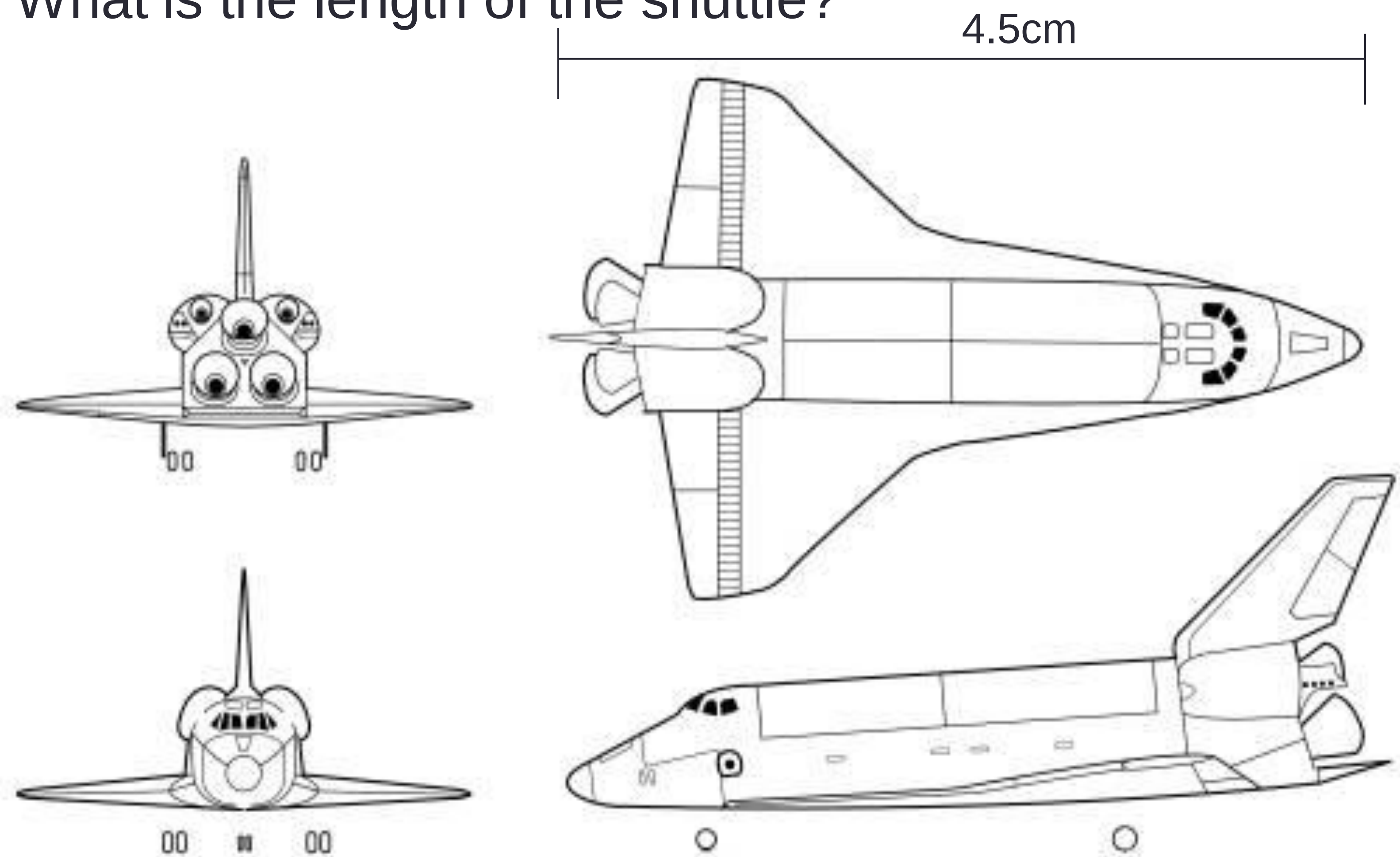
Concept 5: Scale Drawings

- A scale drawing is the same shape as the drawing it represents. The dimensions of the drawing will have the same ratio as the dimensions of the object. The scale is the ratio comparing the drawing and object.



Example 3

- The scale of the drawing to the actual rocket is 1:10,000. What is the length of the shuttle?



Example 4

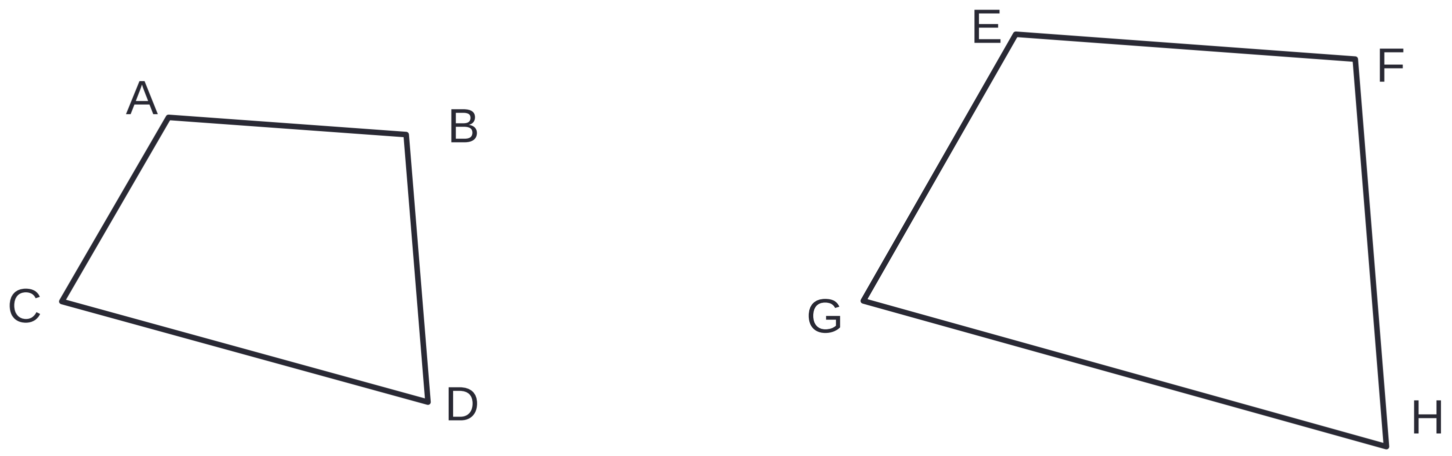
- You buy a 3-D scale model of the Reunion Tower in Dallas, TX. The actual building is 560 feet tall. Your model is 10 inches tall, and the diameter of the dome on your scale model is about 2.1 inches. What is the diameter of the dome and how many times as tall as your model is the actual building?

USE SIMILAR POLYGONS

“Things could be worse. Suppose your errors were counted and published every day, like those of a baseball player.” –Anon.

Concept 6: Similar Polygons

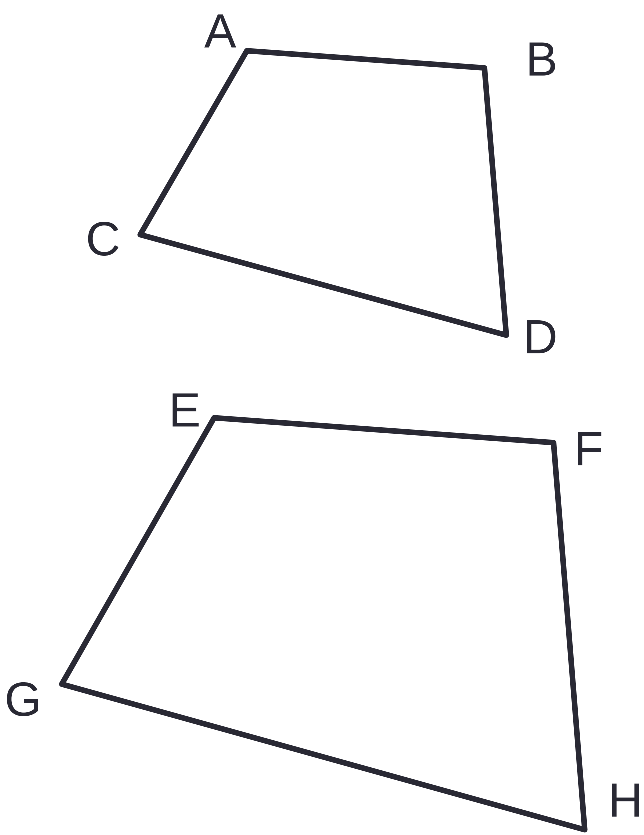
- Polygons that have congruent corresponding angles are similar. Similar means same shape but different size.



$$ABDC \text{ is similar to } EFHG$$
$$ABDC \sim EFHG$$

Concept 7: Corresponding Parts of Similar Polygons

- Corresponding angles are congruent.
- Corresponding sides are proportional.



Corresponding Angles

$\angle A \cong \angle E$, $\angle B \cong \angle F$, $\angle C \cong \angle G$, and $\angle D \cong \angle H$

Corresponding Sides

$$\frac{AB}{EF} = \frac{BD}{FH} = \frac{CD}{GH} = \frac{AC}{EG}$$

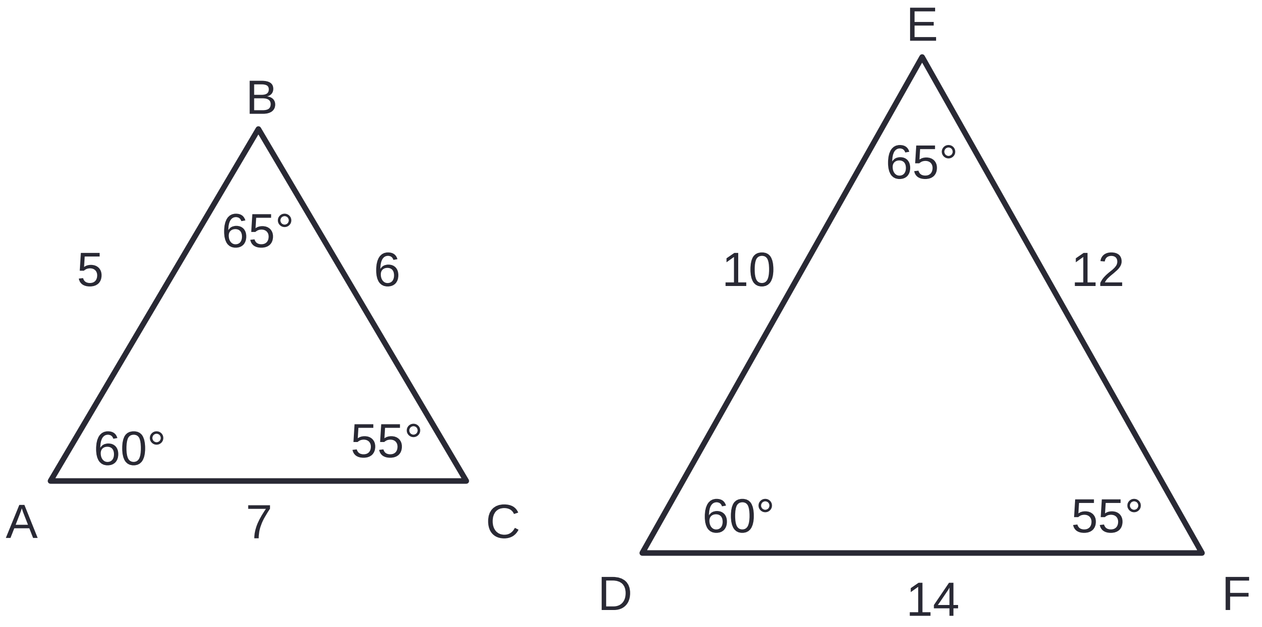
Statement of proportionality

Example 1

- You know that QRST~UVWX. List all pairs of congruent angles. Write the ratios of the corresponding side lengths in a statement of proportionality.

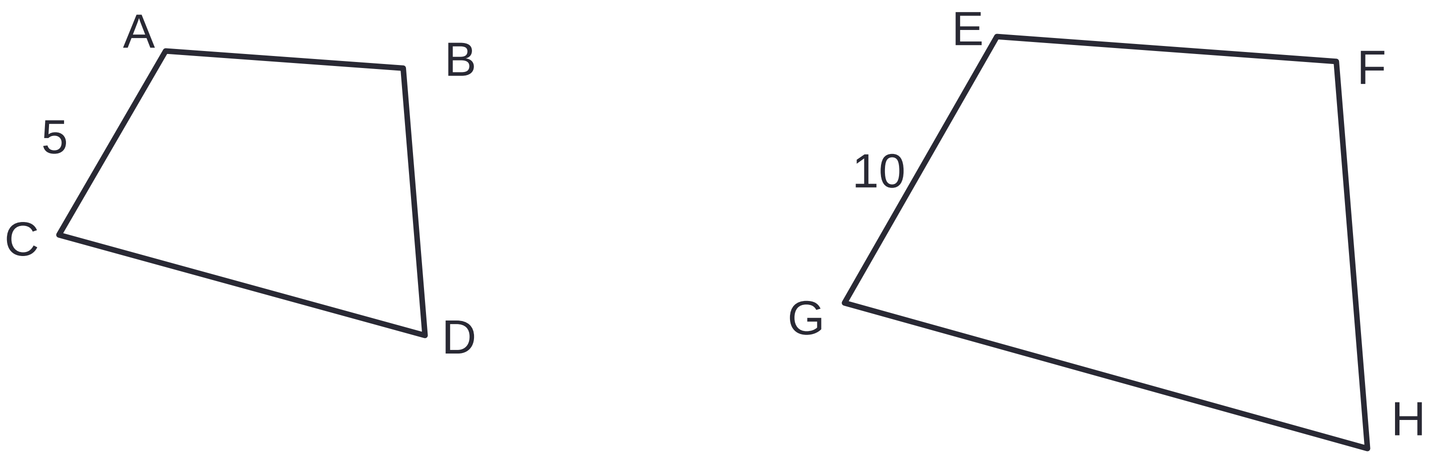
Example 2

- Verify that the two triangles are similar, then write a similarity statement.



Concept 8: Scale Factor

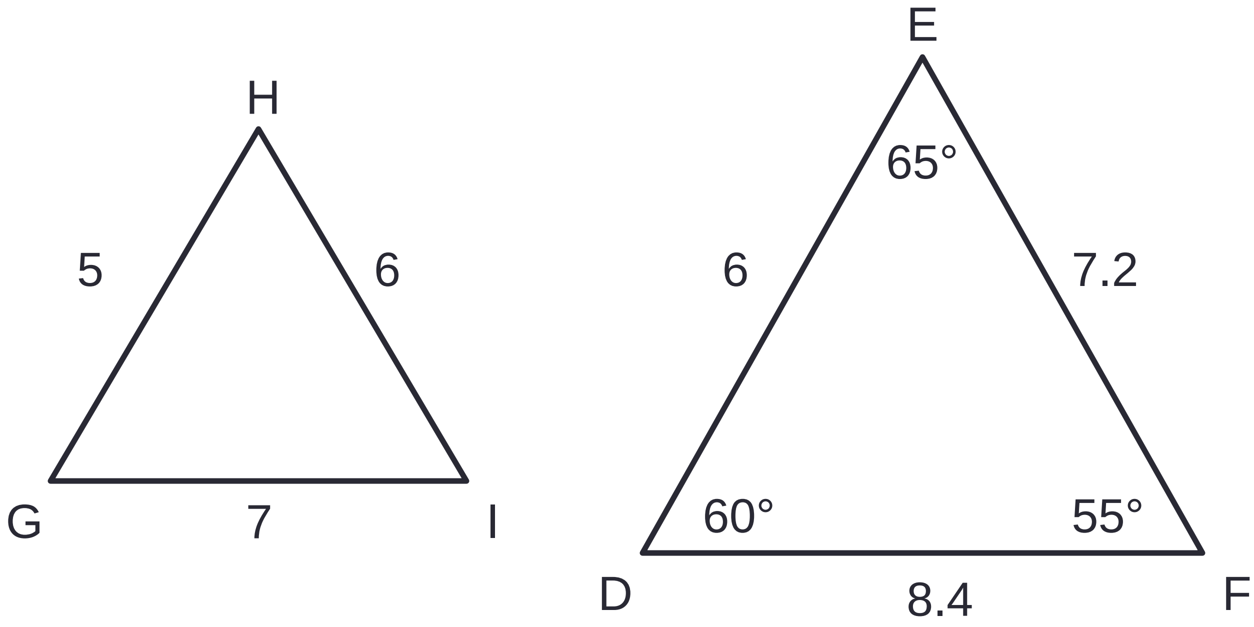
- The ratio of corresponding sides of corresponding triangles is called the scale factor of the polygons.



Scale factor of ABDC to EFHG is 5:10 or 1:2.

Example 3

- Determine the scale factor of the $\Delta DEF\ to\ \Delta GHI$.

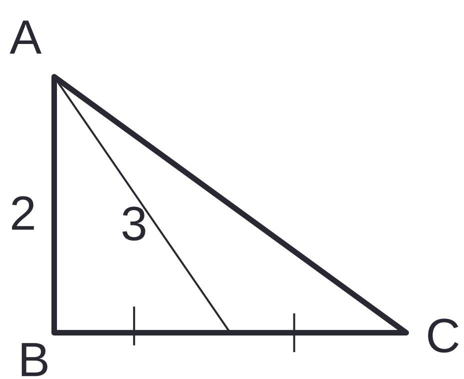


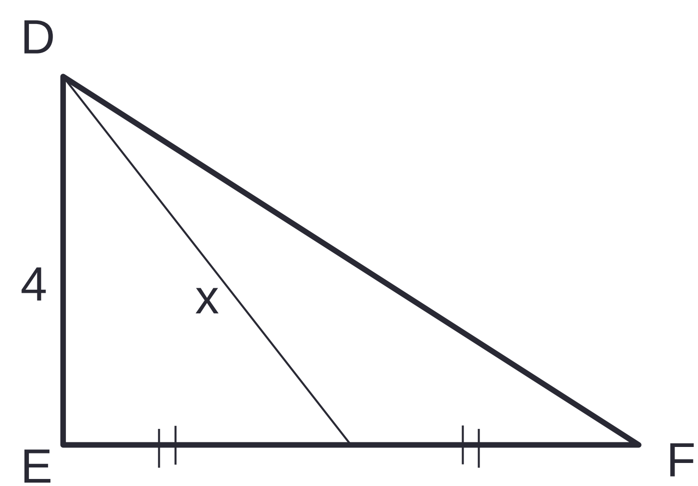
Concept 9: Perimeters of Similar Polygons

- "If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths."

- In fact, the ratio of any corresponding LENGTHS in two similar polygons is equal to the ratios of their corresponding side lengths or scale factor.

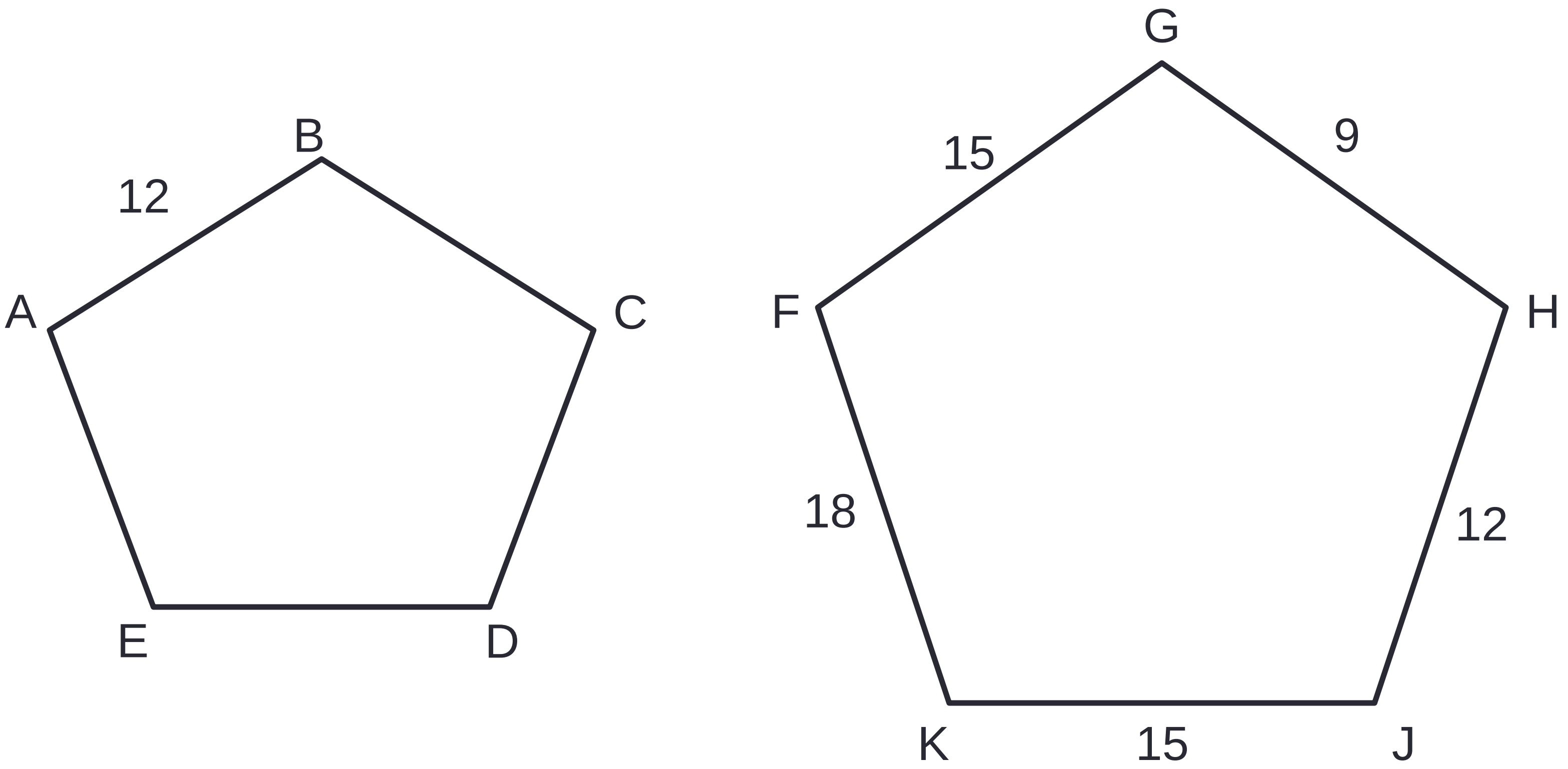
$\Delta ABC \sim \Delta DEF$





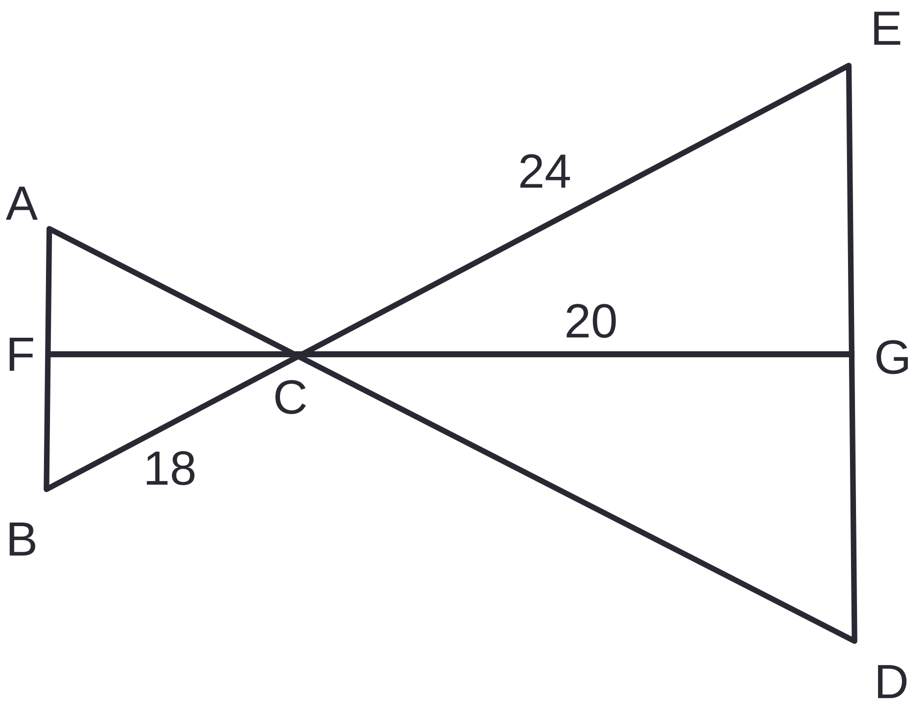
Example 4

- In the diagram, ABCDE~FGHJK. Find the scale factor of the two polygons. What is the perimeter of ABCDE?



Example 5

- In the diagram, $\Delta ABC \sim \Delta DEC$. Find the length of the altitude $\overline{CF}$.

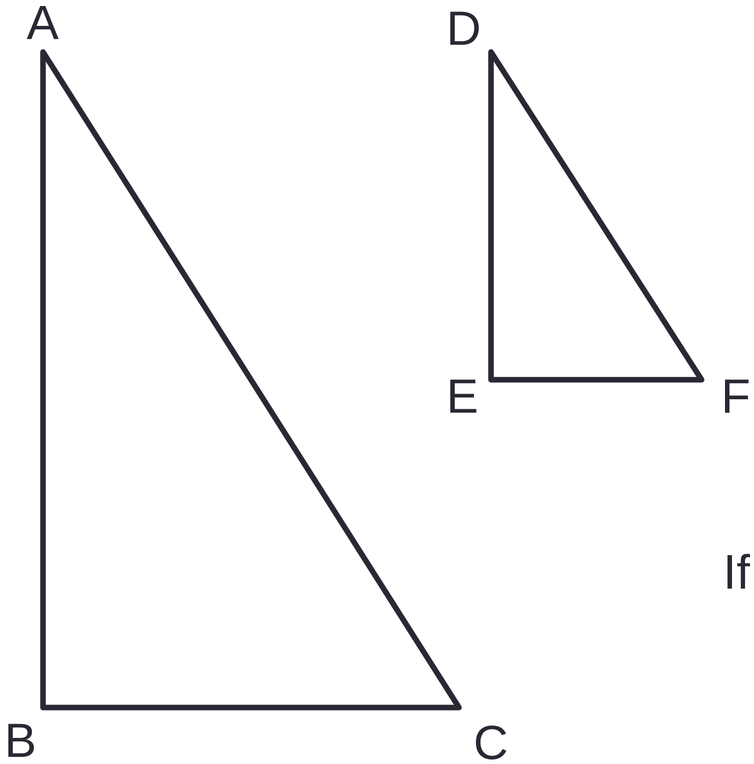


PROVE TRIANGLES SIMILAR BY AA

"The superior man(/woman) blames himself(/herself), the inferior man(/woman) blames others." –Don Shula

Concept 10: AA Similarity Postulate

- If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

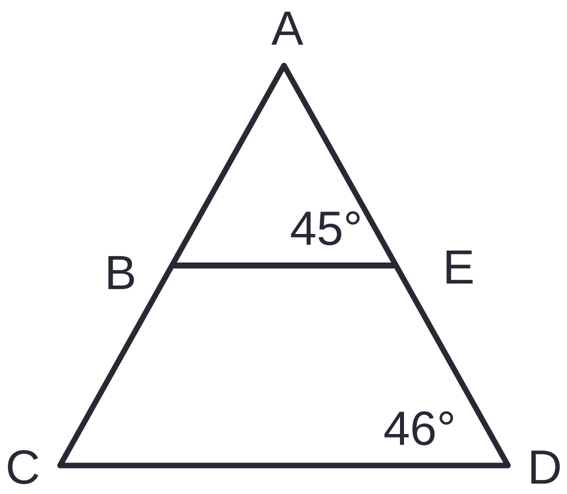


If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\Delta ABC \sim \Delta DEF$.

Example 1

- Determine whether the pair of shapes are similar. If so, write a similarity statement.

1) ΔABE and ΔACD



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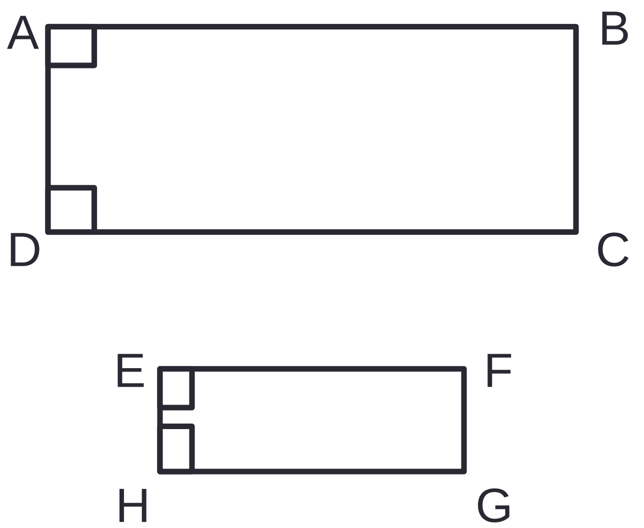

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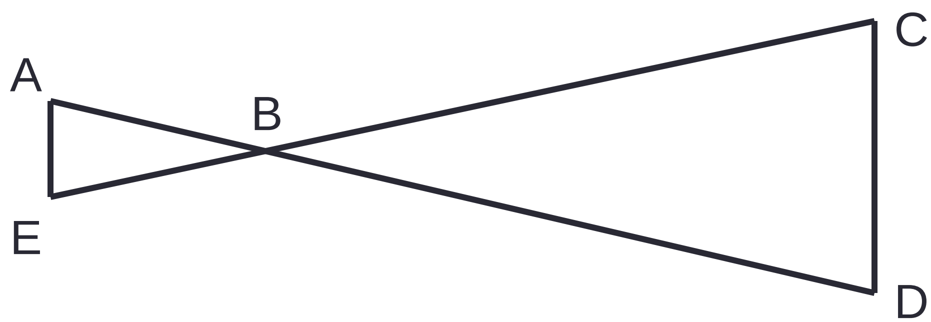
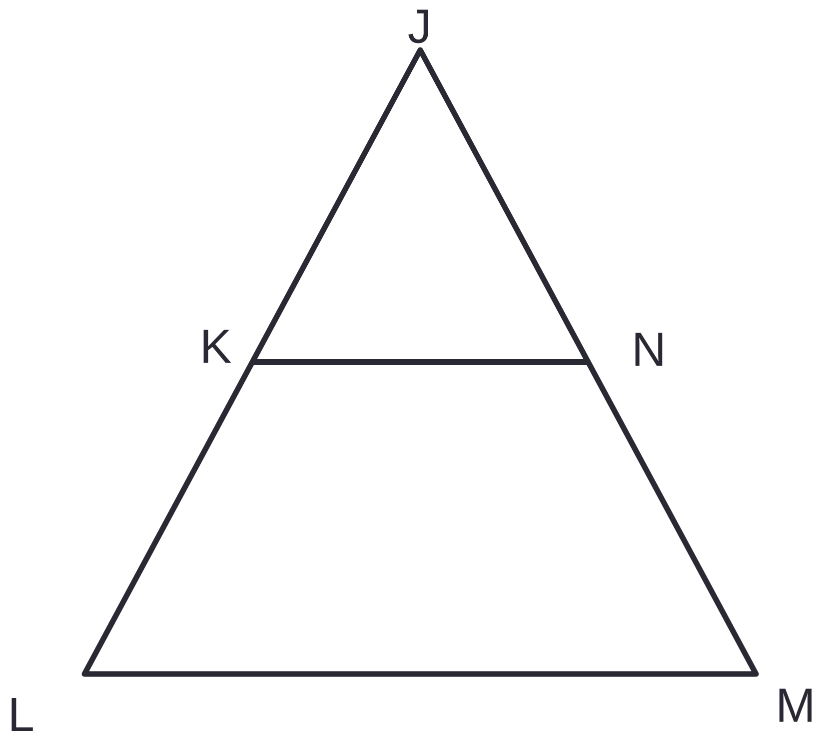


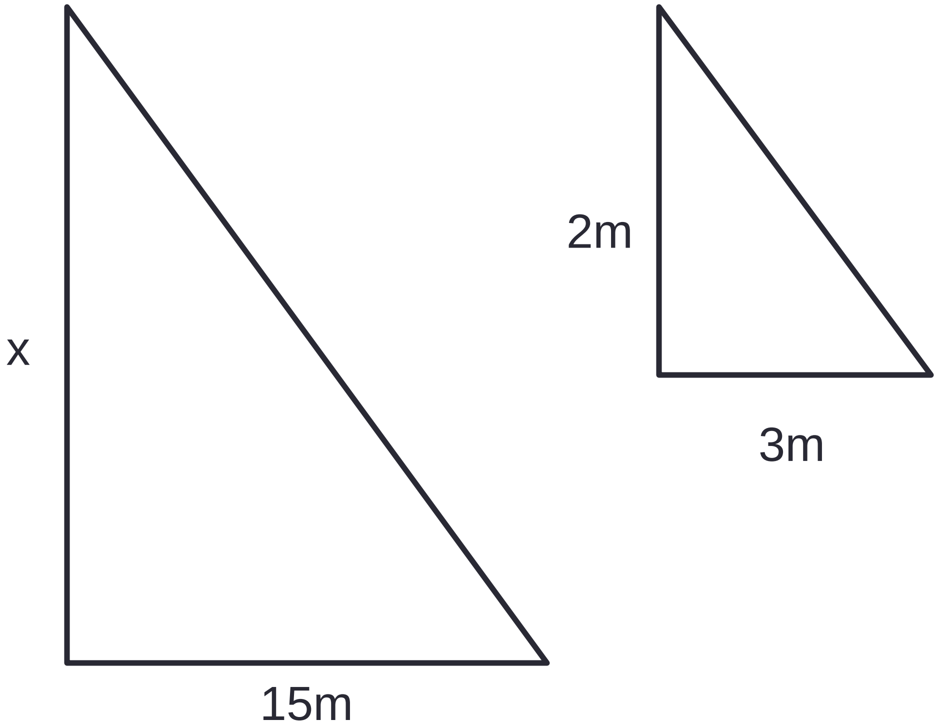
Example 2

- Show that these two triangles are congruent and write a similarity statement and a statement of proportionality.



Concept 11: Using Similar Triangles

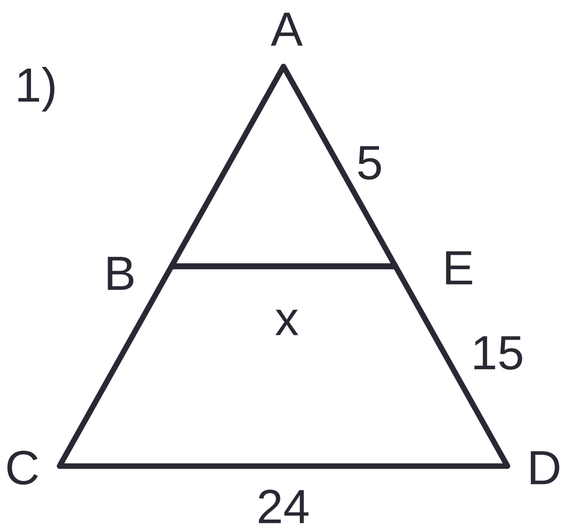
- Knowing two triangles are similar and a few sides lengths you can find measurements indirectly.



Example 3

- Find the value of x.

1)



2) A flagpole casts a shadow that is 50 ft long. At the same time, a woman standing nearby who is five feet four inches tall casts a shadow that is 40 inches long. If the height of the flagpole is x, what is the value of x?

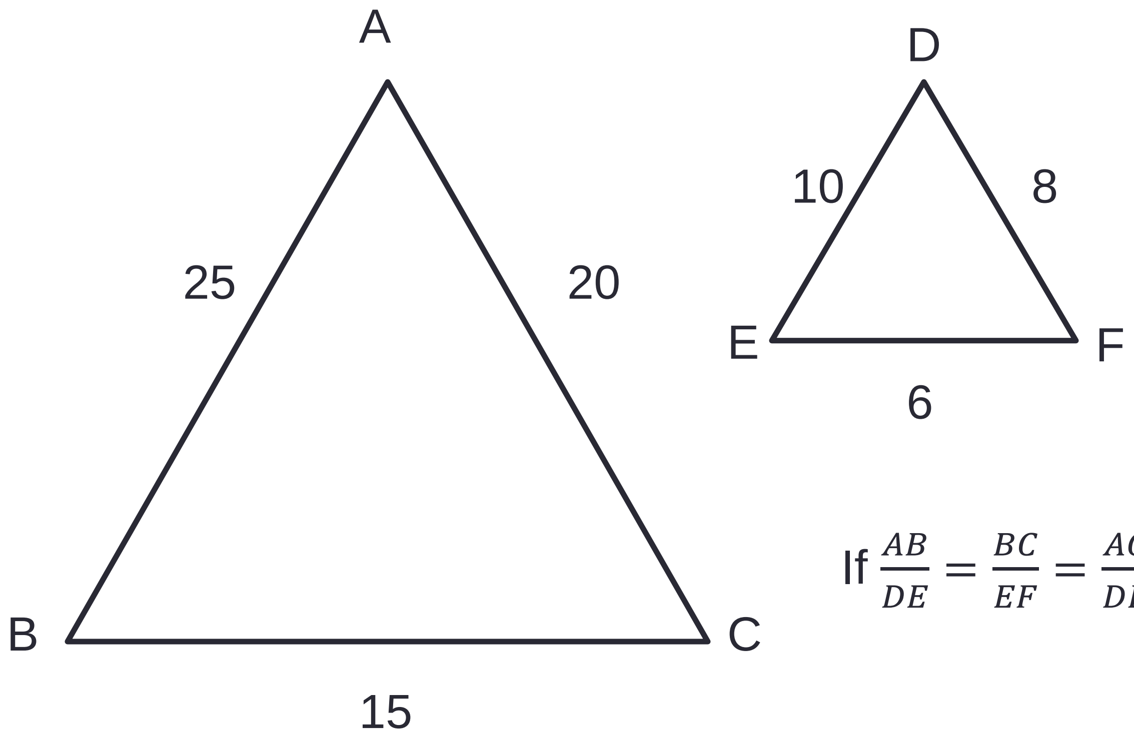
PROVE TRIANGLES SIMILAR BY SSS AND SAS

“That which is bitter to endure may be sweet to remember.”

–Thomas Fuller

Concept 12: SSS Similarity Theorem

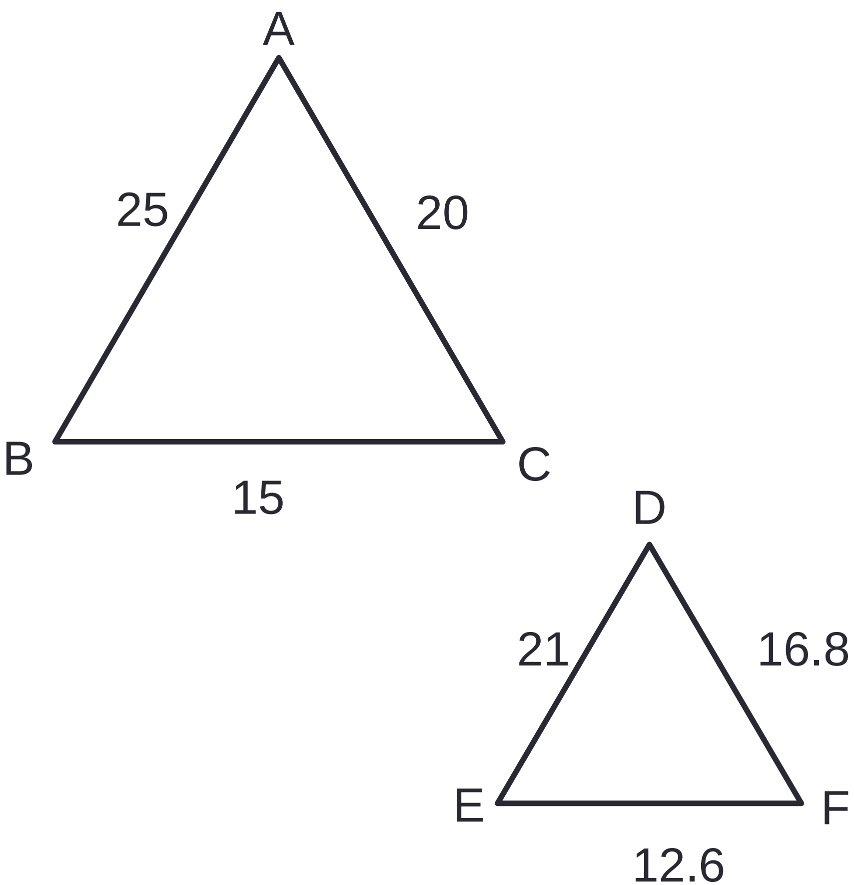
- If the corresponding sides of two triangles are proportional, then the triangles are similar.

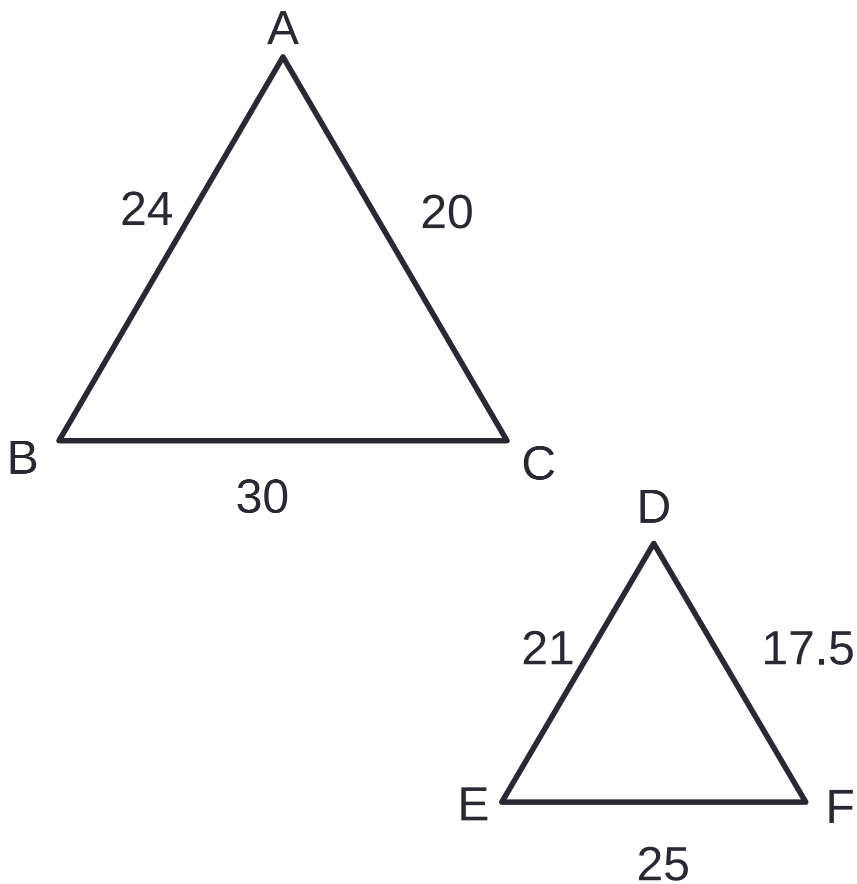


If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\Delta ABC \sim \Delta DEF$.

Example 1

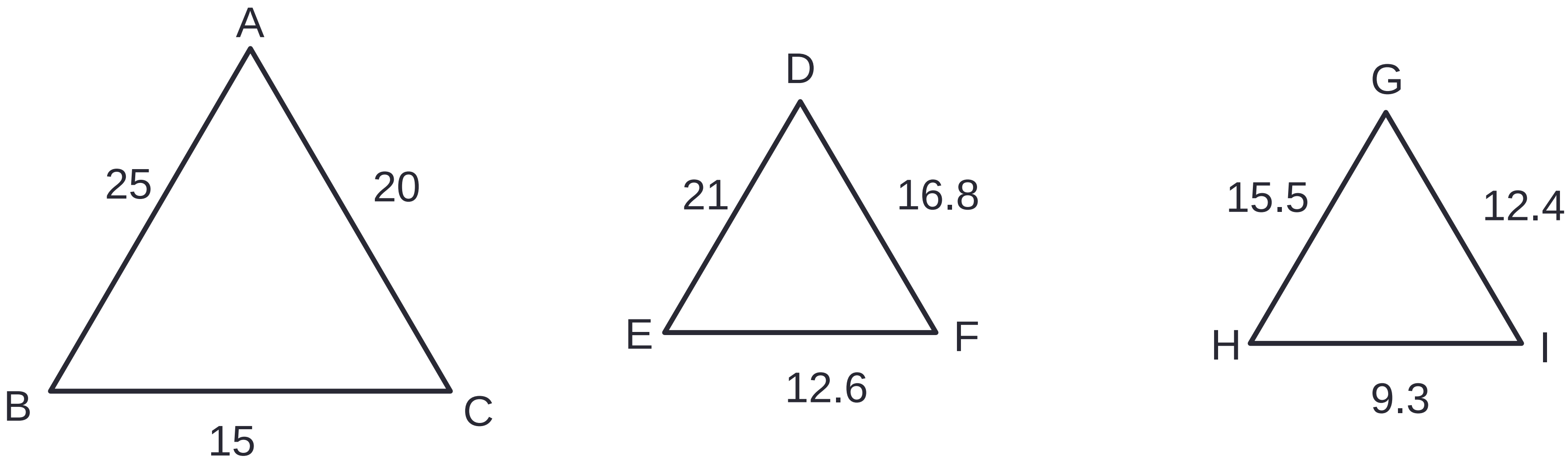
- Determine whether the triangles are similar. If so, write a similarity statement.





Example 2

- Is either ΔABC or ΔDEF similar to ΔGHI?



Example 3

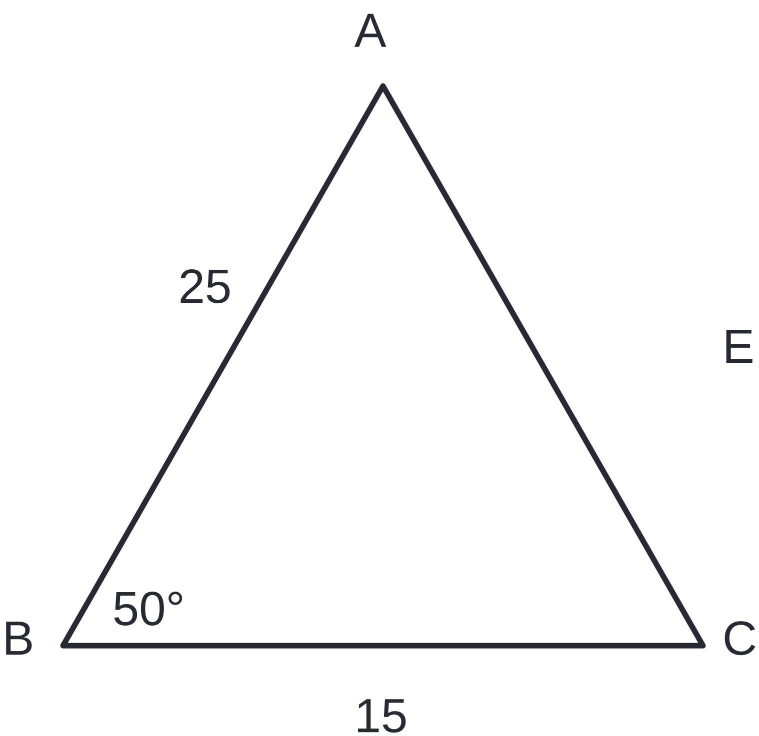
- What additional piece of information do you need to show that $\Delta ABC \sim \Delta DEF$ by SSS?

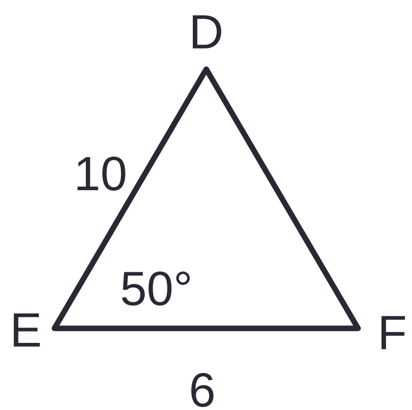
1) It is given that $\frac{AB}{DE} = \frac{BC}{EF}$.

2) It is given that $\frac{DF}{AC} = \frac{DE}{AB}$.

Concept 13: SAS Similarity Theorem

- If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

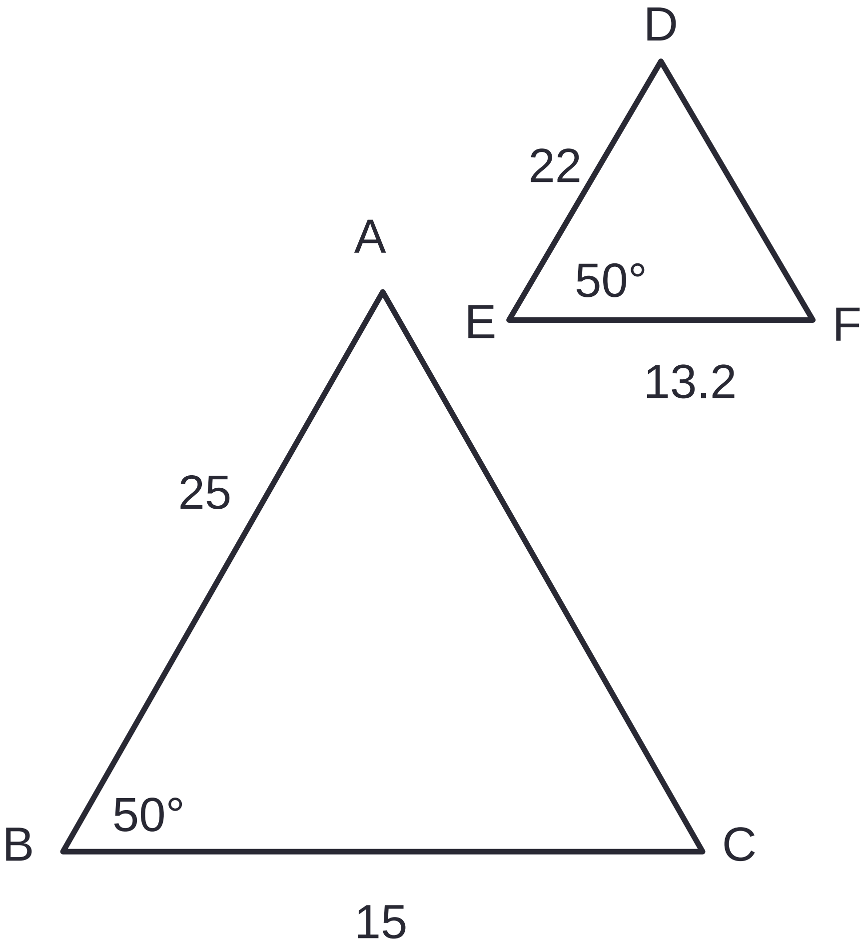


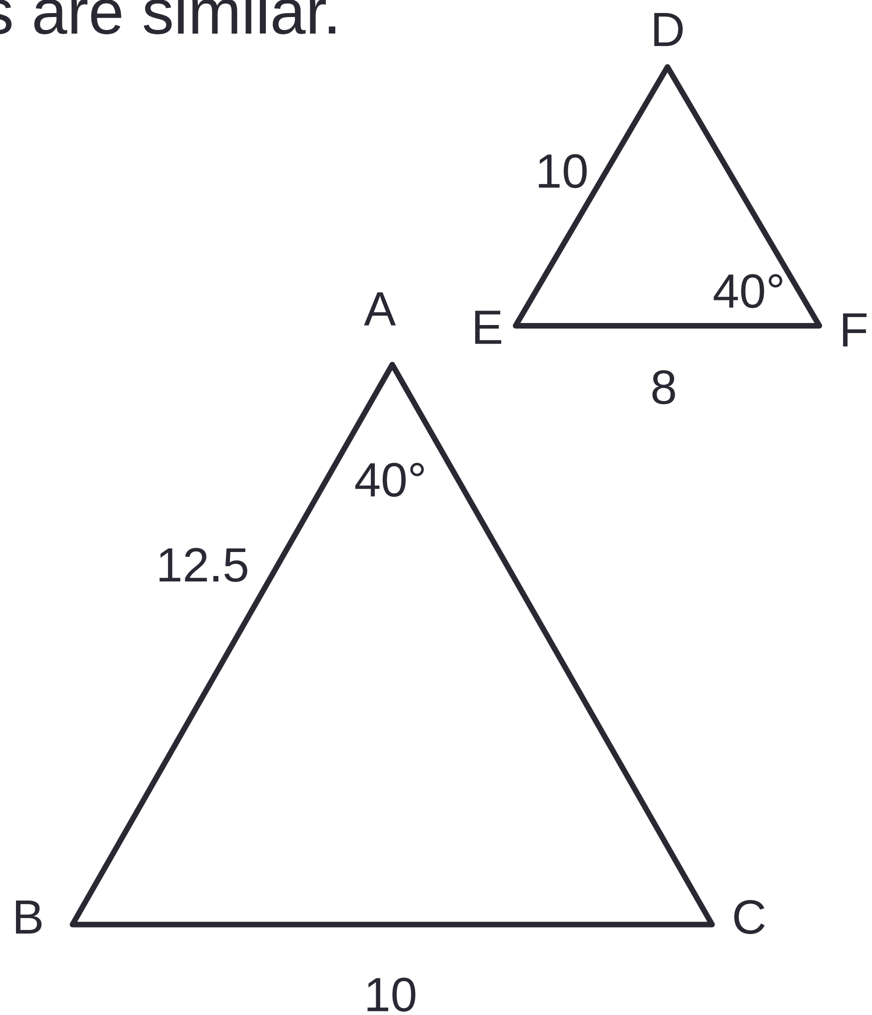


If $\frac{AB}{DE} = \frac{BC}{EF}$ $and\ \angle B \cong \angle E$, then $\Delta ABC \sim \Delta DEF$.

Example 4

- Determine whether the triangles are similar.





Example 5

- Sketch the triangles using the given description. Explain whether the two triangles can be similar.

1) In ΔABC, m∠A=55° and m∠B=45°. In ΔDEF, m∠D=55° and m∠F=80°.

2) The side lengths of ΔGHI are 5, 6, and 7. The side lengths of ΔJKL are 6, 7.2, and 8.4.

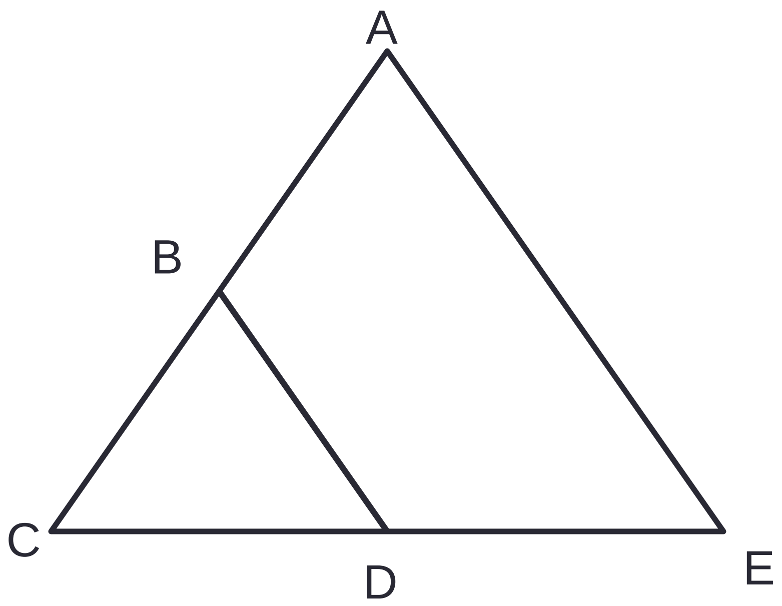
3) In ΔMNP, MN=25, NP=20, and m∠N=65°. In ΔQRS, QR=21, RS=16.8, and m∠R=55°.

USE PROPORTIONALITY THEOREMS

"Memory is the thing you forget with." – Alexander Chase

Concept 14: Triangle Proportionality Theorem

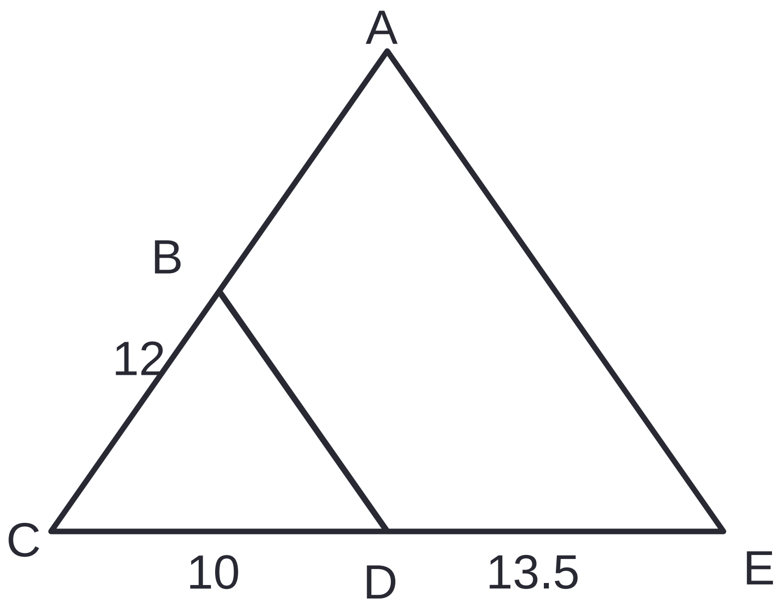
- If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

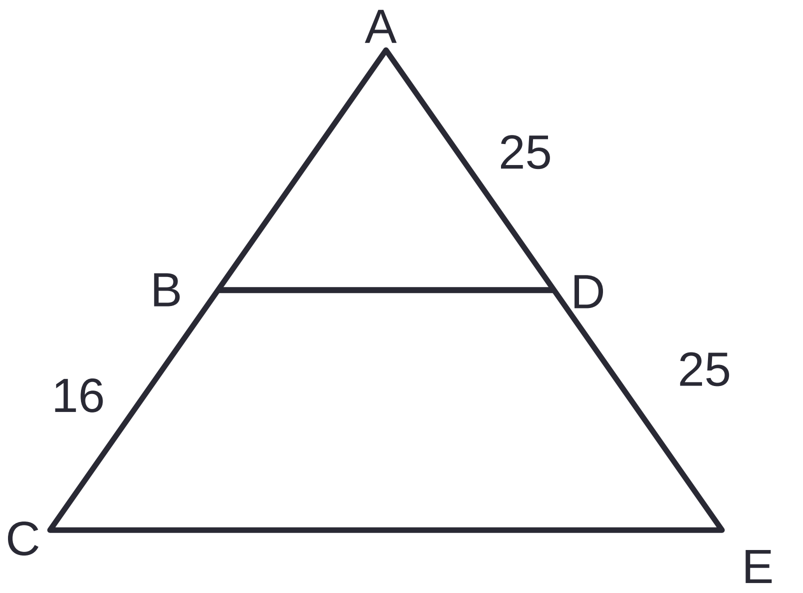


$$\frac{AB}{BC} = \frac{ED}{DC}$$

Example 1

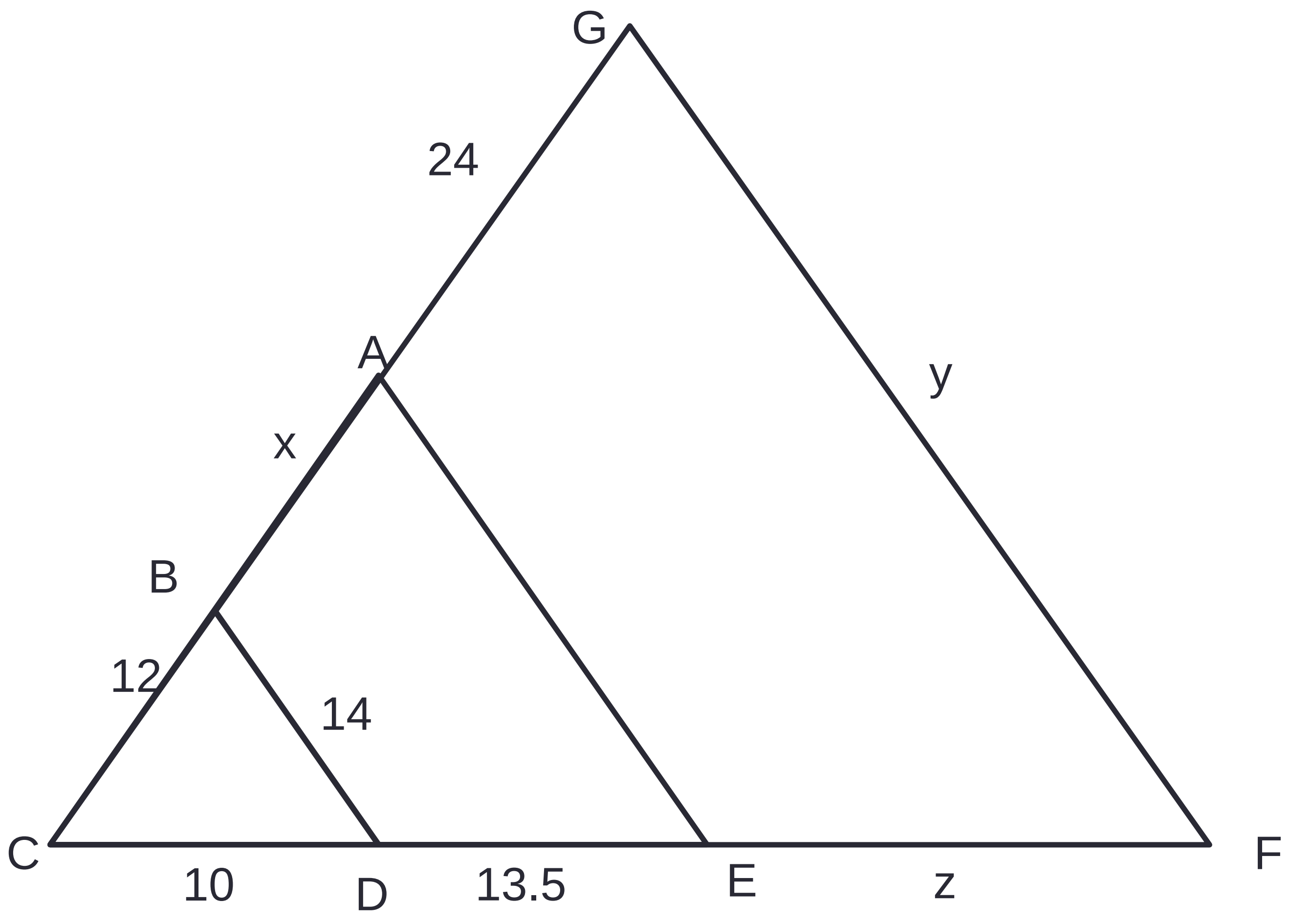
- Find the length of segment AB.





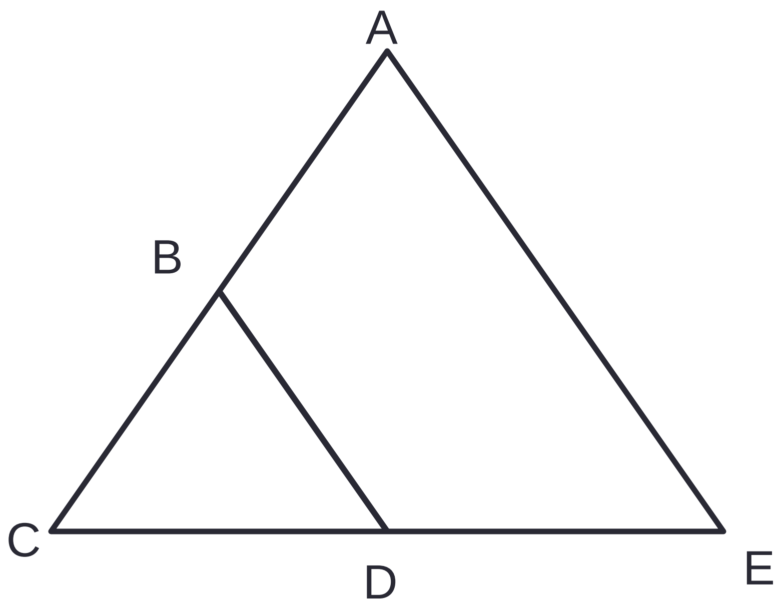
Example 2

- Find the value of each variable.



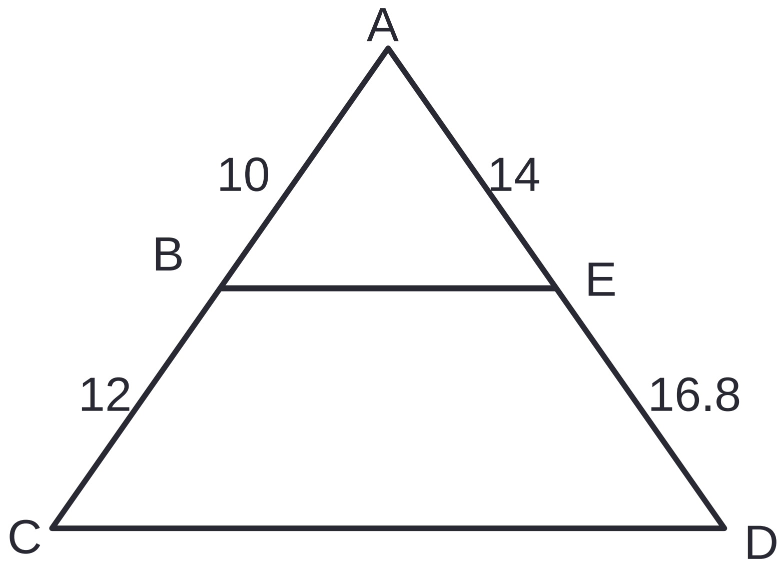
Concept 14: (Converse to the) Triangle Proportionality Theorem

- If a line divides two sides of a triangle proportionally, then it is parallel to the third side.



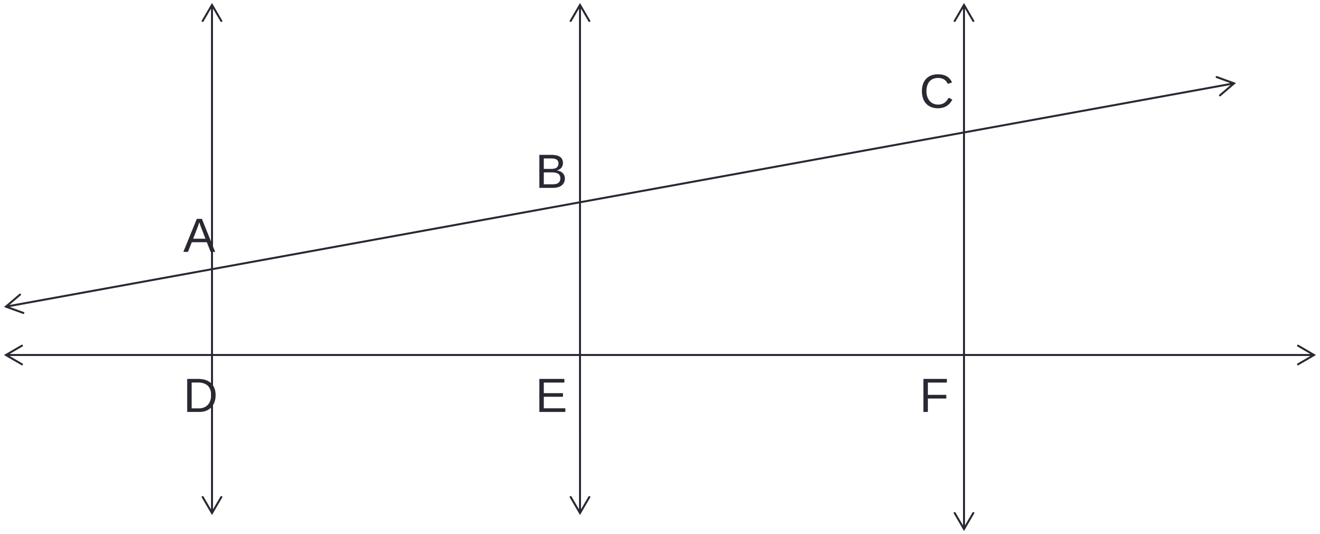
Example 3

- Determine whether $\overline{BE} || \overline{CD}$. Explain your reasoning



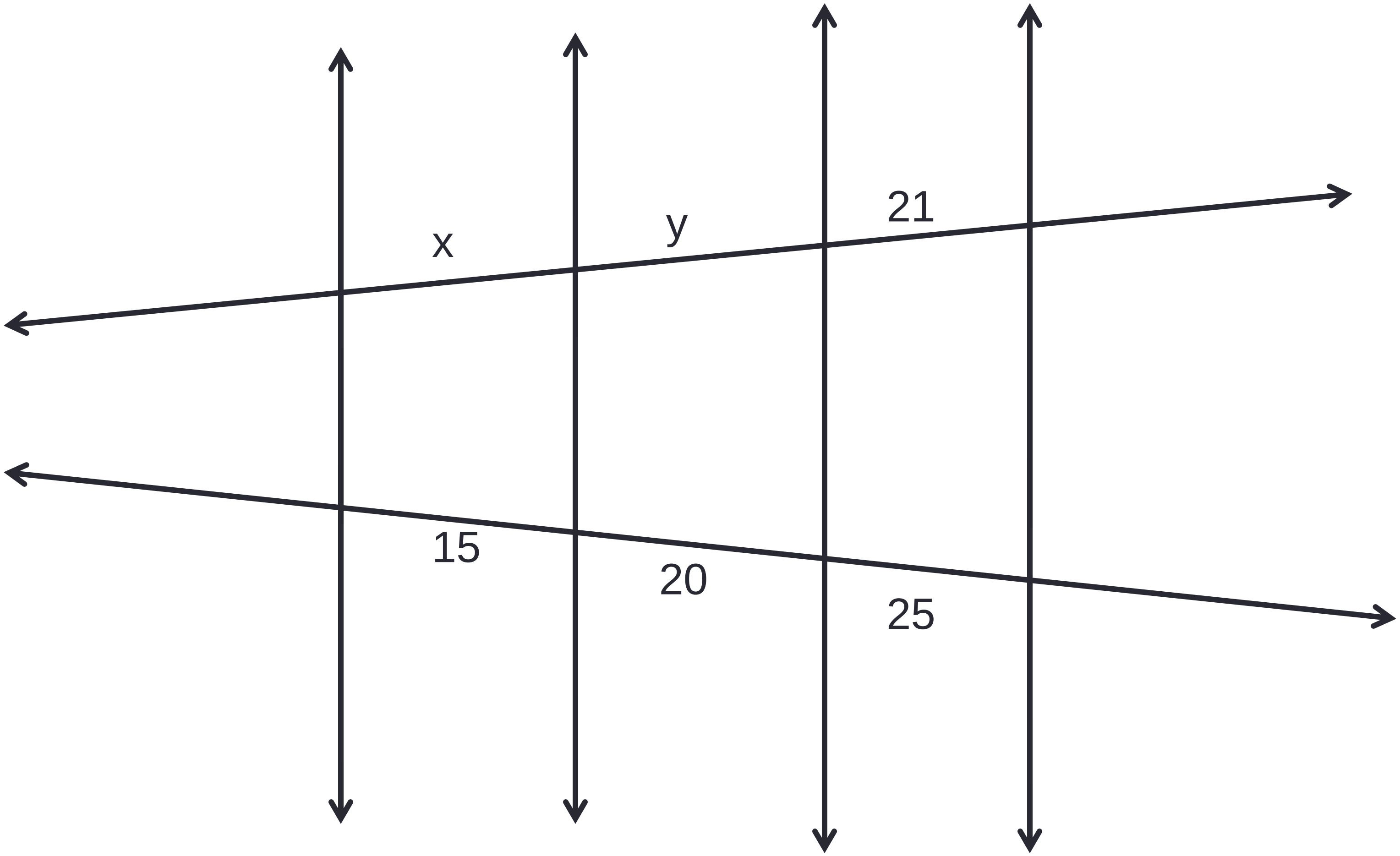
Concept 15: Theorem 6.6

- If three parallel lines intersect two transversals, then they divide the transversals proportionally.



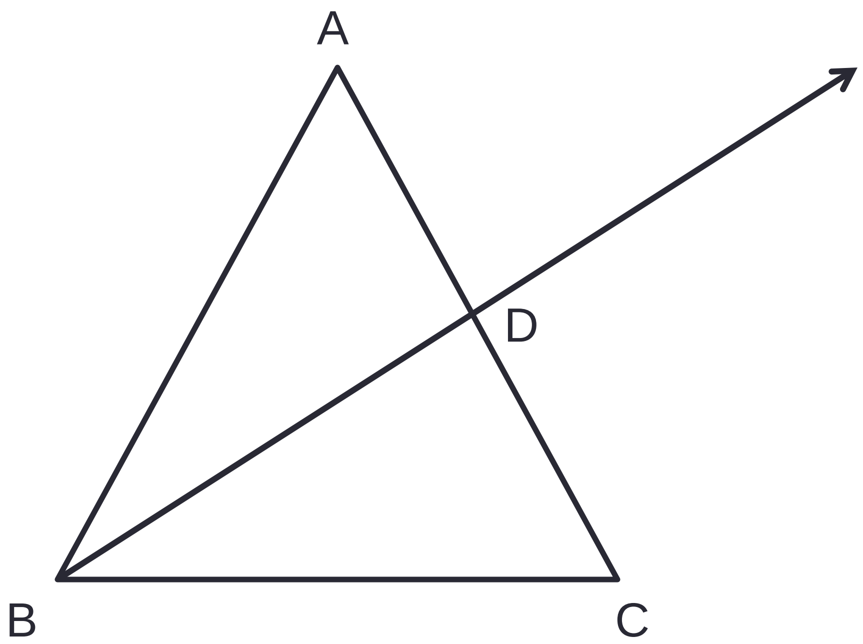
Example 4

- Find the value of x and y.



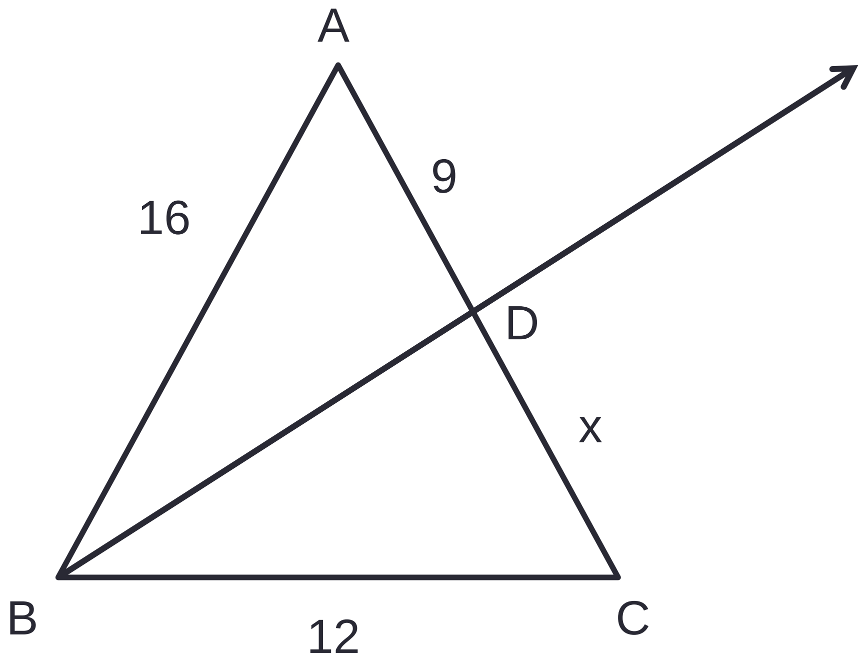
Concept 16: Theorem 6.7

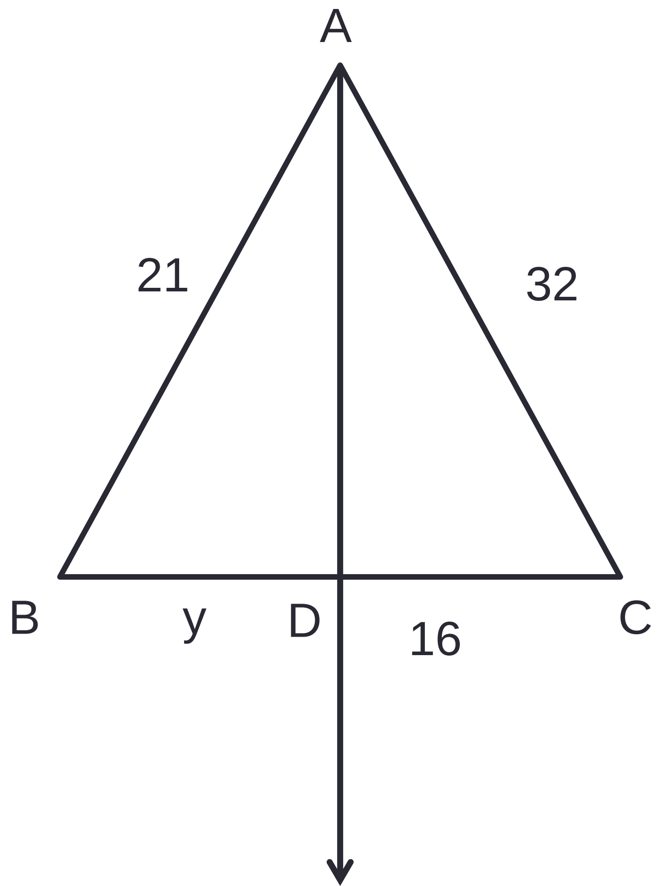
- If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.



Example 5

- Find the value of the variable.





Example 6

- Find the value of each variable.